Solving Matching Problems in Unions of Non-Disjoint Equational Theories

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Matching problems appear naturally in the definition of rewriting. Indeed, for performing a basic rewrite step, one has to solve an equation between the left-hand side of a rule (the *pattern*) and a term (*the subject*) in which the instantiation of variables is forbidden. This special case of equation, where one side can be considered as ground, is called match-equation. Many declarative programming languages use rewriting and equational rewriting as operational semantics. These languages enable the programmer to write rule-based specifications/programs involving operators which, for the sake of expressivity, may satisfy some structural axioms like associativity, commutativity, identity, idempotency, etc... Consequently, it is crucial to develop matching algorithms for equational theories. Moreover, since different equational theories must be implemented in the same framework, we face the problem of combining matching for unions of equational theories possibly sharing function symbols.

The combination problem for the union of non-disjoint equational theories has been already studied for an adequate notion of shared free constructors, and a result on the combination of matching algorithms is known [3]. More recently, some new results have been obtained for non-free constructors, in the context of combining decision procedures for the satisfiability problem [7] and for the word-problem [1, 2, 4]. In this talk, we study the possibility of extending results on the combination of matching algorithms [6], in order to allow shared non-free constructors but also shared non-constructor symbols.

The talk will be organized as follows. First, we present a particularly interesting equational theory T_E, which can be seen as a specific non-disjoint union of two given equational theories T and E. For this particular theory, a combined T_E-matching algorithm can be easily derived from matching algorithms known for T and E, whilst T_E-matching with additional free function symbols is as difficult as E-unification. This non-disjoint union T_E has been used to prove the existence of a theory for which matching is decidable whilst matching with additional free symbols is not [5]. Despite this negative result, we want now to generalize the techniques used for this particular case in order to obtain a way to construct unions of theories where matching problems can still be solved using matching algorithms available in component theories. Then, we compare this approach with the results we could expect by reusing, in the context of matching, the notion of non-free constructors introduced initially for the satisfiability problem and for the word-problem. Finally, as a related work, we report the approach investigated in [8] for solving matching problems thanks an extension of the Nelson-Oppen combination procedure for the satisfiability problem in unions of non-disjoint theories.

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