Approximating E-Unification

Christopher Lynch and Barbara Morawska

Department of Mathematics and Computer Science Box 5815,
Clarkson University, Potsdam, NY 13699-5815, USA,
E-mail: clynch@clarkson.edu, morawskb@clarkson.edu **

Equational logic commonly arises in deductive problems. A particularly important problem is to decide whether there is a substitution that makes a given goal true in an equational theory, or to find a set of substitutions that will generate all possible solutions. This it the problem of *E-unification* [1]. The problem arises in automated deduction, formal verification, type inference and many other areas of computer science. Often deductions are performed modulo a given equational theory, and calls to an E-unification procedure are made constantly through a deductive process.

Unfortunately, E-unification is undecidable in general, even for the problem of only deciding whether a solution exists. However, even when it is decidable, it may have very high complexity. Therefore, it would be useful to have some way of reducing the number of calls to the procedure. One way of doing this would be to approximate E-unification. By this we mean to use an efficient algorithm for deciding E-unification that is complete but not sound. This means that if it says that the E-unification problem is not satisfiable, then we can be sure of the answer. If it says it is satisfiable, we must then run a sound E-unification procedure to determine if that answer is correct. A deductive procedure could then call the approximating algorithm before calling the real algorithm, and avoid many unnecessary calls to the real procedure.

In this paper, we give such an approximating E-unification algorithm. The running time of the algorithm is linear if the equational theory is considered constant, and cubic if the equational theory is considered as part of the input. Our algorithm will approximate any E-unification problem for any equational theory.

What we exactly do in this paper is to show that E-unification is decidable for equational theories such that each equation has no repeated variables and the goal has no repeated variables, with the complexity given above. This class of theories contains all ground equations, but it is larger than that. It is a somewhat restrictive class of theories and has not been studied. However, it is especially interesting for the purpose of approximating E-unification.

The way the approximation works is to take each equation in the goal and the equational theory and rename the variables so that no variable occurs more than once. If this new E-unification problem is unsatisfiable, then the original one is also. In many cases this gives a helpful result, but not in all cases, such as collapse theories.

In order to create the decision procedure, we first create a sound and complete goal-directed inference procedure for the case where the equational theory is linear and the goal contains no repeated variables. The inference rules given for this procedure are similar to the inference rules for Syntactic Theories [4, 6, 7], but in our case we do not require that the theory is syntactic. An interesting feature of this inference system is that we can prove the completeness with Eager Merging. The problem of proving completeness with Eager Merging (or Eager Variable Elimination) is an open problem for many inference

** This work was supported by NSF grant number CCR-9712388 .

systems [2, 3, 5, 8]. Since so little is known about it, it is interesting to have a completeness result with Eager merging, even for such a restricted inference system. The only procedure known to us where Eager Merging has been proved to preserve completeness is given in [11]. Eager Merging is important in practice, because it gives some determinism to a don't-know nondeterministic procedure.

In the conclusion of the paper, we show that our approximation technique is not only useful for E-unification. It is also useful for Logic Programming, where we must decide if there is a substitution that can be applied to the goal so that the goal follows from the given Horn Clauses. We get similar nice complexity results for this case. Our techniques have some similarities to what is done for Local Theories [10], but they are only concerned with universal goals, and we can cover existential goals.

The full version of the paper is at [9].

References

1. F. Baader and T. Nipkow. *Term Rewriting and All That.* Cambridge, 1998.
2. J. Gallier and W. Snyder. A general complete E-unification procedure. In *RTA 2*, ed. P. Lescanne, LNCS vol. 256, 216-227, 1987.
3. J. Gallier and W. Snyder. Complete sets of transformations for general E-unification. In *TCS*, vol. 67, 203-260, 1989.
4. C. Kirchner. Computing unification algorithms. In *Proceedings of the First Symposium on Logic in Computer Science*, Boston, 200-216, 1990.
5. C. Kirchner and H. Kirchner. *Rewriting, Solving, Proving.* `http://www.loria.fr/~ckirchne/` , 2000.
6. C. Kirchner and F. Klay. Syntactic Theories and Unification. In *LICS 5*, 270-277, 1990.
7. F. Klay. Undecidable Properties in Syntactic Theories. In *RTA 4*,ed. R. V. Book, LNCS vol. 488, 136-149, 1991.
8. C. Lynch and B. Morawska. Goal Directed E-Unification. To appear at RTA 2001.
9. C. Lynch and B. Morawska. Approximating E-Unification `http://www.clarkson.edu/~clynch/papers/approx_full.ps/`, 2000.
10. D. McAllester. Automated Recognition of Tractability in Inference Relations. In *Journal of the ACM*, vol.40(2) , pp. 284-303, 1993.
11. A. Middeldorp, S. Okui, T. Ida. Lazy Narrowing: Strong Completeness and Eager Variable Elimination. In *Theoretical Computer Science* 167(1,2), pp. 95-130, 1996.